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NONSTEADY-STATE DRAINAGE FROM POROUS MEDIA

By Marvin E. Jensen,[1] M. ASCE, and R. John Hanks[2]

INTRODUCTION

Drainable porosity or specific yield is a major parameter involved in all land drainage problems. Most solutions of nonsteady-state drainage problems assume abrupt drainage of the pore space as the water table is lowered.[3,4,5,6,7] In many nonsteady-state solutions the drainable porosity is also assumed to be constant, i.e., independent of both depth to the water table and time, even though this is recognized as incorrect.[8] Luthin and Worstell[9] demonstrated

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[1] Research Investigations Leader, Water Management, Northwest Branch, Snake River Conservation Research Center, Kimberly (Twin Falls), Idaho.

[2] Research Investigations Leader, Moisture Conservation, Northern Plains Branch, Fort Collins, Colo.

[3] Bouwer, H., and van Schilfgaarde, J., "Simplified Method of Predicting Fall of Water Table in Drained Land," Transactions, American Society of Agricultural Engineers, Vol. 6, 1963, pp. 288-291, 296.

[4] Dumm, L. D., "New Formula for Determining Depth and Spacing of Subsurface Drains in Irrigated Lands," Agricultural Engineering, Vol. 35, 1954, pp. 726-730.

[5] Ligon, J. T., Johnson, H. P., and Kirkham, D., "Unsteady-State Drainage of Fluid From a Vertical Column of Porous Material," Journal of Geophysical Research, Vol. 67, 1962, pp. 5199-5204.

[6] Luthin, J. N., "The Falling Water Table in Tile Drainage—II: Proposed Criteria for Spacing Tile Drains," Transactions, American Society of Agricultural Engineers, Vol. 2, 1959, pp. 44-45.

[7] Youngs, E. G., "The Drainage of Liquids From Porous Materials," Journal of Geophysical Research, Vol. 65, 1960, pp. 4025-4030.

[8] van Schilfgaarde, J., "Approximate Solutions to Drainage Flow Problems," Drainage of Agricultural Lands, American Society of Agronomy Monograph, Vol. 7, 1957, pp. 79-112.

[9] Luthin, J. N., and Worstell, R. V., "The Falling Water Table in Tile Drainage—A Laboratory Study," Proceedings, Soil Science Society of America, Vol. 21, 1957, pp. 580-584.

with a large sand-filled tank that the assumption of a fixed drainable pore space leads to incorrect solutions of drainage problems. The errors involved in this assumption are considered in detail by Childs.[10]

A more recent solution to the nonsteady-state drainage problem considered drainable porosity as a variable determined by evaluating the volume of water discharged from a field drain system in relation to the volume of soil dewatered.[11,12] This technique combined the effects of water table depth, drainage rate, and time. A detailed evaluation of the variation in drainable porosity as a function of depth to the water table under equilibrium conditions is presented by Taylor.[13] Taylor also presented a laboratory procedure for evaluating the drainable porosity, by lowering the water table by successive increments and allowing the soil column to approach equilibrium at each position. As much as 3 weeks to 4 weeks were required to obtain values for drainable porosity as a function of water table depth using sand and glycerine. Equilibrium methods such as these may overestimate drainable porosity by a factor of two. Field measurements of effective drainable porosity using outflow methods usually are more reliable for design purposes, because both time and depth to the water table affecting drainable porosity are involved.[13]

Drainable porosity under nonsteady-state conditions is dependent primarily on the hydraulic conductivity, which is a nonlinear function of fluid content, and hydraulic gradient. Hydraulic gradient is affected by the nonlinear relationship between fluid content and fluid pressure for a given porous medium and the boundary conditions.

The extremely low values of hydraulic conductivity that occur in agricultural soils after the large pores have drained are largely responsible for the observed slow drainage characteristics of these materials. King[14] observed that water drainaged from a soil column for more than 2 yr. Nixon and Lawless[15] observed drainage and translocation of moisture during an 8-month period. Miller[16] observed drainage from a 4-ft sandy, loam profile for 60 days after irrigation. Slow drainage from soil profiles also implies that "specific yield" as used in hydrological studies is not a constant, but is a function of time in deep profiles.

There are many applications for information on factors affecting or influencing the drainage rate from agricultural soils. For example, evapotranspiration or consumptive use measurements by soil sampling techniques can involve large errors due to slow drainage when evapotranspiration rates are

[10] Childs, E. C., "The Non-Steady-State of the Water Table in Drained Land," Journal of Geophysical Research, Vol. 65, 1960, pp. 780-782.

[11] Dumm, L. D., "Transient-Flow Concept in Subsurface Drainage: Its Validity and Use," Transactions, American Society of Agricultural Engineers, Vol. 7, 1964, pp. 142-146, 151.

[12] Dumm, L. D., and Winger, R. J., Jr., "Subsurface Drainage System Design for Irrigated Area Using Transient-Flow Concept," Transactions, American Society of Agricultural Engineers, Vol. 17, 1964, pp. 147-151.

[13] Taylor, G. S., "Drainable Porosity Evaluation From Outflow Measurements and Its Use in Drawdown Equations," Soil Science, Vol. 90, 1960, pp. 338-343.

[14] King, F. M., Soil Management, Orange Judd, New York, N. Y., 1914.

[15] Nixon, P. R., and Lawless, G. P., "Translocation of Moisture With Time in Unsaturated Soil Profiles," Journal of Geophysical Research, Vol. 65, 1960, pp. 655-661.

[16] Miller, D. E., "Moisture Retention in Synthetic Soil Profiles," 1961 (personal communication).

low.[17,18] As refinements in techniques for predicting evapotranspiration evolve, corrections for drainage between irrigations will be needed to predict accurately total irrigation water requirements. These corrections will require the application of hydraulic principles, and knowledge of the hydraulic properties of porous media.

A typical example of the nature of observed drainage rate from a heavily irrigated soil was presented by Ogata and Richards.[19] Drainage rates were related to the thickness of the soil layer in question, and inversely related to some function of time.

BACKGROUND

Fluid drainage from porous media involves liquid flow in unsaturated porous media. An early analysis of unsaturated horizontal flow was presented by Buckingham.[20] Richards[21] presented a detailed theoretical analysis of this problem incorporating both pressure gradient and gravity effects. Darcy's law and the continuity equation were combined, resulting in a differential equation in cartesian coordinates for the general case of unsaturated fluid flow. The Richards equation for the one-dimensional case reduces to

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K \frac{\partial H}{\partial z}\right] \quad \text{. . . . .} \quad (1)$$

in which θ = the volumetric fluid content, t = time, K = unsaturated hydraulic conductivity, H = hydraulic head ($H = P_w/\gamma_f + z = h + z$), z = the vertical coordinate, P_w = the pressure of the wetting fluid and γ_f = the weight of fluid per unit volume. A detailed derivation and discussion of this equation is presented by Liakopoulos.[22,23] The nonlinear relationships between K and θ, and θ and P_w generally restrict solutions of Eq. 1 to numerical methods. Day and Luthin[24] have shown that solving Eq. 1 by manual numerical methods is long and laborious if acceptable accuracy is desired. Therefore, accurate solutions are essentially restricted to high-speed computers.

[17] Robins, J. S., Pruitt, W. O., and Gardner, W. H., "Unsaturated Flow of Water in Field Soils and Its Effect on Soil Moisture Investigations," Proceedings, Soil Science Society of America, Vol. 18, 1954, pp. 344-347.

[18] Wilcox, J. D., "Rate of Soil Drainage Following an Irrigation—II: Effects on Determination of Rate of Consumptive Use," Canadian Journal of Soil Science, Vol. 40, 1960, pp. 15-27.

[19] Ogata, G., and Richards, L. A., "Water Content Changes Following Irrigation of Bare-Field Soil that is Protected From Evaporation," Proceedings, Soil Science Society of America, Vol. 21, 1957, pp. 355-356.

[20] Buckingham, E., "Studies on the Movement of Soil Moisture," Soils Bulletin No. 38, U. S. Dept. of Agriculture, 1907.

[21] Richards, L. A., "Capillary Conduction of Liquids Through Porous Systems," Physics, No. 1, 1931, pp. 318-333.

[22] Liakopoulos, A. C., "Theoretical Solution of the Unsteady Unsaturated Flow Problems in Soils," Bulletin of the International Association of Scientific Hydrology, Vol. 10, 1965, pp. 5-39.

[23] Liakopoulos, A. C., "Derivation of Differential Equation Governing Simultaneous Flow of Liquids and Gases Through Porous Media," Transactions, American Society of Agricultural Engineers, Vol. 8, 1965, pp. 210-215.

[24] Day, P. R., and Luthin, J. N., "A Numerical Solution of the Differential Equation of Flow for a Vertical Drainage Problem," Proceedings, Soil Science Society of America, Vol. 20, 1956, pp. 443-447.

Approximate solutions that produce results of sufficient accuracy and can be solved quickly and easily are adequate for many nonsteady-state drainage problems. Youngs,[7] using the analogy of capillary tubes and constant drainable porosity, presented the following equation for a vertical drainage problem in a gravitational field with the plane of zero fluid pressure at the base of the column, i.e.,

$$\frac{Q}{Q_\infty} = 1 - e^{-q_0 t/Q_\infty} \qquad (2)$$

in which Q = the total outflow in time t, Q_∞ = the total outflow as $t \to \infty$, and q_0 = the initial flow rate. Eq. 2 considers only the first term in a series solution. The mathematical approximations leading to Eq. 2 underestimate Q/Q_∞, whereas the assumption of an equal quantity of fluid drainage for an equal advance of the saturated front overestimates Q/Q_∞. Eq. 2 would be expected to describe drainage while the saturated front is falling. The derivation by Youngs is similar to the opposite conventional method of computing rate of capillary rise, and a derivation of horizontal flow using tubes of different diameters, presented by Lambe.[25]

Gardner[26] derived a similar approximate solution based on the assumption of constant diffusivity and an adjustment for the capillary fringe based on a solution by Miller and Elrick.[27] Only the first term in the series solution is important after a short time resulting in

$$\frac{Q}{Q_\infty} = 1 - \frac{8}{\pi^2}\, e^{-(\alpha_1)^2 \overline{D} t/L^2} \qquad (3a)$$

in which $\overline{D} = \overline{K}\, L^2/Q_\infty$ . . . . . (3b)

L = the length of the column, $\overline{K}$ = the average hydraulic conductivity between the surface of the column and the saturated zone ($\overline{D}/L^2 = \overline{K}/Q_\infty$), and $(\alpha_1)^2$ is obtained from the ratio of impedance of the fringe, z_1, to that of the remainder of the column, $(L - z_1)$. The values of $(\alpha_1)^2 = \pi^2/4$ when $z_1 = 0$. A similar derivation was presented by Fujioka and Kitamura.[28] Selection of the appropriate value for $\overline{K}$ may limit the usefulness of Eq. 3(a). Gardner evaluated Eq. 3(a) by comparison with experimental data, but selected a value of $\overline{K}$ to fit the data. In practice, a value of $\overline{K}$ must be predetermined. These approximate solutions will be compared with experimental data and the numerical solutions using predetermined hydraulic parameters for the porous materials involved.

The equilibrium curve in Fig. 1 represents a typical relationship between fluid pressure head, h, ($h = -z$) and fluid content, θ, for a homogeneous porous medium following the drainage cycle. The fluid pressure at the surface of the column must approach a value numerically equal to z_1 before rapid desatura-

[25] Lambe, T. W., "Capillary Phenomena in Cohesionless Soils," Transactions, ASCE, Vol. 116, 1951, pp. 401-432.

[26] Gardner, W. R., "Approximate Solutions of a Non-Steady-State Drainage Problem," Proceedings, Soil Science Society of America, Vol. 26, 1962, pp. 129-132.

[27] Miller, E. E., and Elrick, D. E., "Dynamic Determination of Capillary Conductivity Extended for Non-Negligible Membrane Impedance," Proceedings, Soil Science Society of America, Vol. 22, 1958, pp. 483-486.

[28] Fujioka, Y., and Kitamura, T., "Approximate Solution of a Vertical Drainage Problem," Journal of Geophysical Research, Vol. 69, 1964, pp. 5249-5255.

tion occurs. A sudden change in fluid pressure at the surface when drainage first begins has been verified experimentally by Luthin and Miller.[29] Lambe[21] postulated that this change in fluid pressure would take place almost instantly. Desaturation near the surface of the column would initially be rapid, with the value of θ corresponding to the fluid pressure-fluid content relationship illustrated between points 1 and 2 in Fig. 1. Beyond point 2 an incremental change in θ requires an increasingly larger change in h. Also, the relative hydraulic conductivity, k_r, of the unsaturated porous medium decreases rapidly during the initial desaturation period, as illustrated in Fig. 2 ($k_r = K/K_0$,

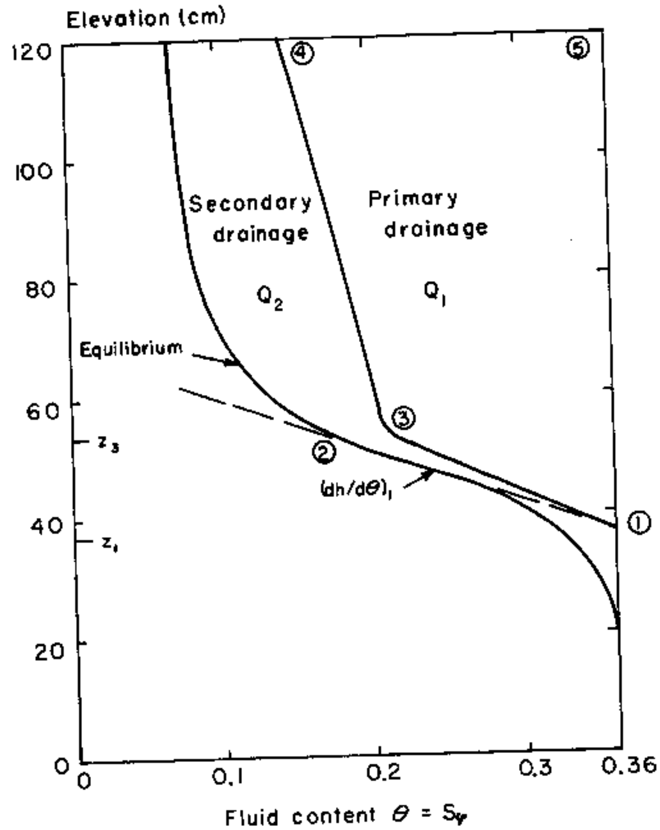

FIG. 1.—TYPICAL EQUILIBRIUM RELATIONSHIP BETWEEN ELEVATION AND FLUID CONTENT, AND AN ILLUSTRATION OF PRIMARY AND SECONDARY DRAINAGE

in which K_0 = saturated hydraulic conductivity). Thus, as a saturated column drains, rapid desaturation of a layer would be expected until the fluid pressure, h, decreased to a critical value illustrated in Fig. 2 by h_c. One would also expect the drainage rate, q, and the rate of change in the drainage rate, dq/dt, to be large until $h \rightarrow h_c$ throughout most of the column, after which q and dq/dt would become much smaller. Beyond this point Youngs' equation would not be expected to represent continued slow drainage from a porous medium column.

[29] Luthin, J. N., and Miller, R. D., "Pressure Distribution in Soil Columns Draining into the Atmosphere," Proceedings, Soil Science Society of America, Vol. 17, 1953, pp. 329-333.

Vertical drainage could be divided into two categories defined as primary and secondary drainage:

Primary drainage.—This represents the period of drainage when the fluid rapidly drains from the larger soil pores and the rate of drainage is controlled by the position of the saturated front at z. The quantity of fluid drained would be approximately proportional to the fall of the saturated front. The period of time involved corresponds to the rapid fall of the saturated front at z from $t = 0$ or $z = L$ until z approaches the equilibrium position at z_1. The latter point appears to correspond to the value of $\tau \approx 1.0$ in Youngs'[7] equation in which $\tau = q_0 t/Q_\infty$. Youngs equation fit his experimental data well for $0 < \tau \approx 1.0$, and for Q/Q_∞ up to about 0.63.

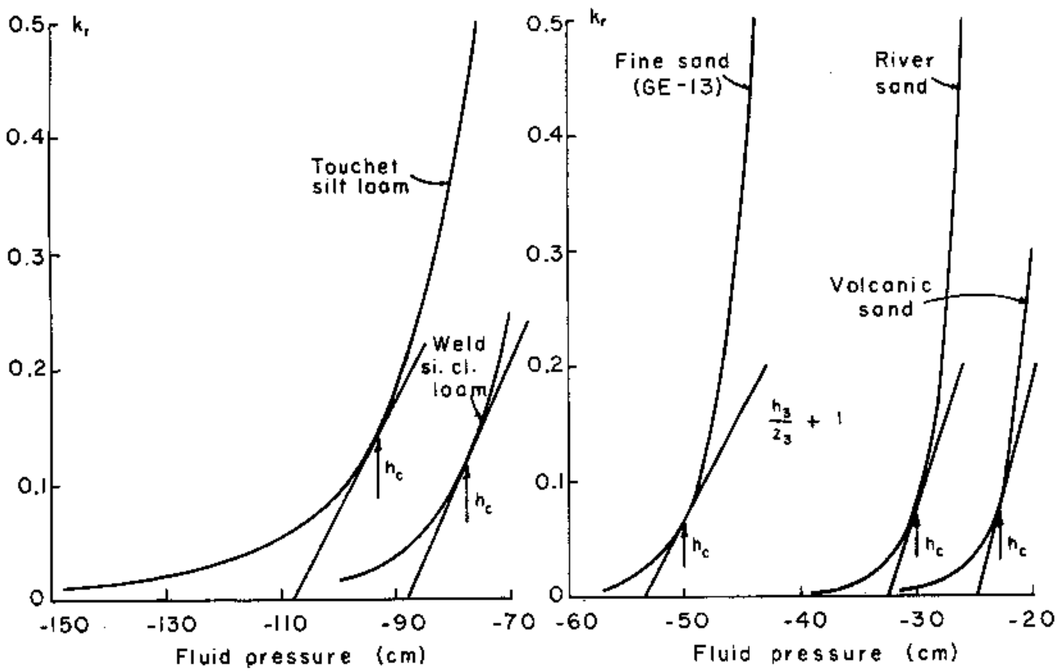

FIG. 2.—RELATIVE PERMEABILITY OR RELATIVE HYDRAULIC CONDUCTIVITY, k_r, AS A FUNCTION OF FLUID PRESSURE HEAD, h

Secondary drainage.—This represents the period of drainage when the saturated front has approached the equilibrium position at z_1, and slow drainage from the layers above the transition zone now controls the drainage rate independent of the movement of the saturated front. The drainage rate would no longer be proportional to dz/dt, and Youngs' equation would no longer be expected to represent the drainage rate accurately.

Procedures for estimating the volume of primary drainage, Q_1, the transition time, t_1, the elevation of the plane represented by z_3, and the absence of a transition in flow regime if the relative hydraulic conductivity at z_3 remained greater than the critical value represented by $h_3/z_3 + 1$ in Fig. 2 until most of the drainage had occurred, are presented and evaluated in detail elsewhere.[30]

PROCEDURE

The general approach used in this study was: (1) To obtain solutions to nonsteady-state vertical drainage problems by laboratory experiments and numerical methods; and (2) to evaluate simple approximate solutions by comparison with the more exact solutions. The data obtained in the laboratory were used primarily to confirm the numerical solutions obtained using a high-speed computer.

TABLE 1.—SUMMARY OF CONDITIONS FOR LABORATORY EXPERIMENTS

Run	Column		Fluid Used	Porous Medium	
	Length, in centimeters	Diameter, in centimeters		Name	Porosity, ϕ
E-1	126	3.18	Soltrol[a,b]	River sand (screened)	0.40
E-2	136	3.18	Soltrol[a,b]	Volcanic sand	0.38
E-3	126	3.18	Soltrol[a,b]	Fine sand (G.E. -13)	0.39

[a]Soltrol "C" core test fluid. Phillips Petroleum Company, Special Products Division, Bartlesville, Oklahoma. (Specific weight = 0.753 g per cm^3; dynamic viscosity at room temperature = approximately 1.47 centipoise).

[b]Trade names and company names are included for the benefit of the reader and do not imply any endorsement or preferential treatment of the product listed by the U.S. Department of Agriculture.

TABLE 2.—CHARACTERISTIC PARAMETERS FOR THE POROUS MATERIALS USED

Number	Name	Intrinsic perme-ability, k, in square centi-meters	Hydraulic conduc-tivity variation index, η	Pore-size dis-tribution index, λ	Bubbling pressure P_b/γ_f, in cen-timeters	Saturated hydraulic conduc-tivity, K_0, in cen-timeters per day
1	Volcanic sand	16.1	9.0	2.29	16.0	700
2	River sand (screened)	9.2	12.7	5.3	24.0	400
3	Fine sand (G.E. -13)	2.5	14.6	3.7	41.0	108
4	Weld silty clay loam	1.13	7.85	1.62	57.0	49.0
5	Touchet silt loam	0.5	5.6	1.82	75.0	21.7

Laboratory Phase.—Three experiments were conducted in the porous media laboratory at Foothills Campus of Colorado State University, using three different porous materials. A summary of the conditions for each of the experiments is presented in Table 1. The parameters describing the hydraulic

[30]Jensen, M. E., "Nonsteady-State Drainage of Fluid from Porous Media and Drainable Porosity," thesis presented to Colorado State University, at Fort Collins, Colo., in 1965, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

TABLE 3.—RELATION OF FLUID PRESSURE HEAD, h, AND HYDRAULIC CONDUCTIVITY, K, TO VOLUMETRIC FLUID CONTENT, θ

θ	Volcanic Sand		River Sand (screened)		Fine Sand (G.E. - 13)		Weld Silty Clay Loam		Touchet Silt Loam	
	h, in centimeters	K, in centimeters per day	h, in centimeters	K, in centimeters per day	h, in centimeters	K, in centimeters per day	h, in centimeters	K, in centimeters per day	h, in centimeters	K, in centimeters per day
0.05	-185.0	0.0	—	—	—	—	—	—	—	—
0.06	-83.5	0.0005	-150.0	0.0	—	—	—	—	—	—
0.07	-59.5	0.01	-50.0	0.04	-119.0	0.0000065	—	—	—	—
0.08	-48.0	0.07	-42.2	0.36	-88.0	0.00076	—	—	—	—
0.09	-40.0	0.35	-39.3	0.92	-77.8	0.0059	—	—	—	—
0.10	-35.6	1.0	-37.1	1.92	-70.9	0.023	—	—	—	—
0.11	-33.0	1.9	-35.3	4.0	-65.9	0.081	—	—	—	—
0.12	-30.8	3.6	-34.1	6.0	-62.6	0.173	—	—	—	—
0.13	-29.1	5.0	-32.9	9.6	-60.1	0.324	—	—	—	—
0.14	-27.8	9.8	-31.7	15.2	-58.2	0.518	-212.0	0.0029	—	—
0.15	-26.5	12.0	-30.7	24.0	-56.6	0.864	-190.0	0.0048	—	—
0.16	-25.4	20.0	-30.0	32.0	-55.2	1.40	-173.0	0.011	—	—
0.17	-24.5	28.0	-29.4	42.0	-53.9	2.16	-157.0	0.024	-235.0	0.0015
0.18	-23.8	37.0	-28.8	52.0	-52.8	3.02	-145.0	0.046	-212.0	0.030
0.19	-23.1	49.0	-28.3	64.0	-51.8	4.0	-134.0	0.081	-196.0	0.046
0.20	-22.4	70.0	-27.9	80.0	-50.8	5.3	-125.0	0.14	-181.0	0.069
0.21	-21.8	98.0	-27.6	92.0	-49.9	6.9	-118.0	0.23	-166.5	0.080
0.22	-21.2	133.0	-27.3	106.0	-49.1	9.0	-111.0	0.35	-156.5	0.16
0.23	-20.7	168.0	-26.9	122.0	-48.3	11.9	-106.0	0.54	-147.5	0.22
0.24	-20.3	196.0	-26.7	138.0	-47.5	15.7	-101.0	0.74	-140.0	0.29
0.25	-19.9	222.0	-26.4	155.0	-46.7	21.1	-97.0	1.03	-133.0	0.39
0.26	-19.4	245.0	-26.3	172.0	-45.7	28.6	-93.2	1.40	-127.5	0.50
0.27	-19.0	273.0	-26.2	190.0	-44.7	40.8	-89.8	1.86	-123.0	0.63
0.28	-18.5	312.0	-25.9	210.0	-43.5	58.3	-86.5	2.45	-118.0	0.74
0.29	-18.0	354.0	-25.7	228.0	-42.3	72.4	-83.7	3.33	-114.0	0.91
0.30	-17.6	385.0	-25.6	248.0	-40.9	82.0	-81.3	4.02	-110.2	1.09
0.31	-17.1	417.0	-25.4	268.0	-39.4	89.6	-78.8	5.00	-107.0	1.35
0.32	-16.7	441.0	-25.1	288.0	-37.5	97.0	-76.7	6.20	-104.2	1.52
0.33	-16.1	473.0	-24.7	308.0	-35.5	104.0	-75.0	7.30	-101.4	1.80
0.34	-15.5	511.0	-24.2	332.0	-32.8	108.0	-73.3	8.70	-98.8	2.10
0.35	0.0	700.0	-23.7	352.0	-29.0	108.0	-71.7	10.50	-96.4	2.42
0.36	—	—	-22.7	372.0	0.0	—	-70.1	12.25	-94.1	2.82
0.37	—	—	-21.9	384.0	—	—	-68.7	14.8	-92.0	3.23
0.38	—	—	-20.4	394.0	—	—	-67.1	17.6	-90.3	3.67
0.39	—	—	-18.2	400.0	—	—	-65.9	20.6	-88.5	4.23
0.40	—	—	0.0	400.0	—	—	-64.7	23.5	-86.7	4.89
0.41	—	—	—	—	—	—	-63.7	26.1	-85.0	5.58
0.42	—	—	—	—	—	—	-62.7	28.3	-83.5	6.23
0.43	—	—	—	—	—	—	-62.0	29.9	-82.1	6.90
0.44	—	—	—	—	—	—	-61.0	31.8	-80.7	7.60
0.45	—	—	—	—	—	—	-59.7	34.0	-78.7	8.77
0.46	—	—	—	—	—	—	-57.6	38.3	-75.0	11.17
0.47	—	—	—	—	—	—	0.0	49.0	-67.0	15.47
0.48	—	—	—	—	—	—	—	—	0.0	21.70

characteristics of these materials and materials used in the computer programs are summarized in Table 2. The descriptive parameters used are essentially the same as those proposed by Brooks and Corey,[31,32] and are defined as

$$S_e = \frac{S - S_r}{1 - S_r} \quad (4)$$

in which S_r (called residual saturation) is the saturation at which K is assumed to be zero for calculation purposes. Effective saturation and capillary pressure, P_c, are related for $P_c \geq P_b$ as

$$S_e = (P_b/P_c)^\lambda \quad (5)$$

in which λ = the negative slope of S_e versus P_c/γ_t. The term λ is defined as the pore-size distribution index, and P_b is the approximate minimum value of

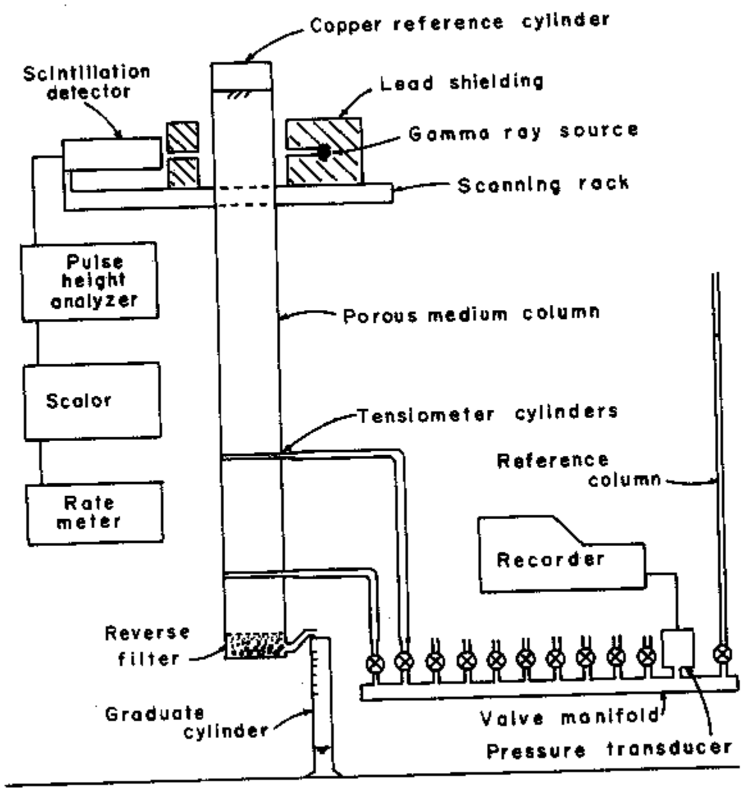

FIG. 3.—SCHEMATIC SKETCH OF THE LABORATORY APPARATUS

P_c on the drainage cycle at which a continuous nonwetting phase exists in the porous medium. The relative hydraulic conductivity, k_r, for the wetting phase is related to capillary pressure for $P_c \geq P_b$ as

[31] Brooks, R. H., and Corey, A. T., "Hydraulic Properties of Porous Media," Colorado State University Hydrology Papers, No. 3, 1964.

[32] Brooks, R. H., and Corey, A. T., "Hydraulic Properties of Porous Media and Their Relation to Drainage Design," Transactions, American Society of Agricultural Engineers, Vol. 7, 1964, pp. 26–28.

$$k_r = (P_b/P_c)^{\eta} \qquad (6)$$

On logarithmic graph paper with S_e and P_c as coordinates, Eq. 5 plots as a straight line with an intercept, $P_c = P_b$ at $S_e = 1.0$. Eq. 6 also plots as a straight line on logarithmic graph paper with k_r and P_c as coordinates with an intercept, $P_c = P_b$ at $k_r = 1.0$. Detailed hydraulic conductivity, and fluid pressure head versus fluid content data, are presented in Table 3. These data were determined independently by steady-state techniques, except for the saturated hydraulic conductivity, K_0. The columns were packed in three segments using a mechanical packer similar to the design by Jackson, et al.[33] Each of the three segments of the column was assembled from 4-cm plain plastic sections and 2-cm spacers or ceramic tensiometer sections. After packing each segment, beginning with the lower segment, they were combined and sealed with a plastic tape to form the full length column containing 126 cm to 136 cm of porous material. All joints were sealed with tape to restrict the position of air entry to the surface of the column. The columns were placed in the retaining rack to obtain dry gamma radiation attenuation values, and then vacuum saturated. After saturation, the saturated gamma radiation attenuation values were obtained before beginning the drainage run.

The gamma ray attenuation technique, as described by van Bavel,[34] Chase,[35] Fergusson and Gardner,[36] and Rawlins and Gardner,[37] was adapted for measurement of fluid content in the column during drainage.

A manifold valve arrangement, a potentiometric pressure transducer, and a millivolt recorder were used to measure the hydraulic head at each of 10 tensiometer positions. A schematic sketch of the laboratory apparatus is presented in Fig. 3. The columns were disassembled after each run to permit an independent check of the final saturation values and the porosity of the column.

Fluid distribution measurements were made continuously, except for short breaks, during the first 9 hr, 7 hr, and 6 hr of experiments E-1, E-2, and E-3, respectively. These measurements were concentrated primarily in the zone of desaturation. Fluid distribution measurements were made periodically during the balance of each experiment. The degree of saturation values obtained using the gamma ray technique were plotted and the specific time values were obtained from a smooth curve fitted to these points. The durations of the laboratory experiments were 1.0 day, 0.8 day, and 2.6 days for E-1, E-2, and E-3, respectively.

Hydraulic head measurements were also made continuously during the first time periods indicated above. The valve manifold was switched to a new position as soon as the recorder trace indicated that the pressure apparatus was in equilibrium with the fluid pressure in the column at that position. The

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[33] Jackson, R. D., Reginato, R. J., and Reeves, W. E., "A Mechanized Device for Packing Soil Columns," ARS 41-52, U. S. Dept. of Agriculture, 1962.

[34] van Bavel, C. H. M., Underwood, N., and Ragar, S. R., "Transmission of Gamma Radiation by Soils and Soil Densiometry," Proceedings, Soil Science Society of America, Vol. 21, 1957, pp. 588-591.

[35] Chase, G. D., and Rabinowitz, J. L., Principles of Radioisotope Methodology, Burgess Publishing Company Minneapolis, Minn., 1962.

[36] Fergusson, H., and Gardner, W. H., "Water Content Measurement in Soil Columns by Gamma Ray Absorption," Proceedings, Soil Science Society of America, Vol. 26, 1962, pp. 11-14.

[37] Rawlins, S. L., and Gardner, W. H., "A Test of the Validity of the Diffusion Equation for Unsaturated Flow of Soil Water," Proceedings, Soil Science Society of America, Vol. 27, 1963, pp. 507-511.

hydraulic head values were then obtained from the millivolt recorder trace and a calibration curve. The calibration curve was checked periodically using the reference column of fluid shown in Fig. 3. The reference column could be raised or lowered to obtain a range in hydraulic head.

Numerical Solutions.—Thirteen numerical solutions were obtained using the five porous materials whose characteristics are summarized in Table 2. A summary of column length and porous materials used for each solution is presented in Table 4.

Total drainage, Q, drainage rate, q, fluid pressure distribution, and column saturation were computed for each of the numerical solutions from $t = 0$

TABLE 4.—SUMMARY OF NUMERICAL SOLUTIONS OBTAINED AND CORRESPONDING LABORATORY EXPERIMENTS

Numerical Solution Number	Column Length, in centimeters	Porous Medium	Corresponding Laboratory Experiment
N-1	126	Fine sand (G.E. - 13)	E-3
N-2	80	Fine sand (G.E. - 13)	——
N-3	50	Fine sand (G.E. - 13)	——
N-4	300	Touchet silt loam	——
N-5	140	Touchet silt loam	——
N-6	100	Touchet silt loam	——
N-7	136	Volcanic sand	E-2
N-8	60	Volcanic sand	——
N-9	30	Volcanic sand	——
N-10	126	River sand (screened)	E-1
N-11	30	River sand (screened)	——
N-12	200	Weld silty clay loam	——
N-13	120	Weld silty clay loam	——

until $Q \rightarrow Q_\infty$. Eq. 1, expressed in terms of fluid pressure head, was the basic equation used in the computer program, i.e.,

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial \theta} \frac{\partial}{\partial z} \left[K \frac{\partial H}{\partial z}\right] \quad \text{(7)}$$

The numerical form of Eq. 7 was the same as that used by Hanks and Bowers.[38] The working equation is

$$\frac{h_i^j - h_i^{j-1}}{\Delta t} = 1/C_i^{j-1/2} \left[\frac{\left(h_{i-1}^{j-1} + h_{i-1}^j - 2\,\Delta z - h_i^{j-1} - h_i^j\right) K_{i-1/2}^{j-1/2}}{2\,(\Delta z)^2}\right.$$

$$\left.\frac{\left(h_i^{j-1} + h_i^j - 2\,\Delta z - h_{i+1}^{j-1} - h_{i+1}^j\right) K_{i+1/2}^{j-1/2}}{2\,(\Delta z)^2}\right] \quad \text{(8a)}$$

in which the subscripts i pertain to specific increments or layers, j superscripts refer to time, Δz in the numerator is the gravitational term, and C is the volumetric differential fluid capacity,

$$C_i^{j-1/2} = (\partial\theta/\partial h)_i^{j-1/2} \quad \text{(8b)}$$

[38] Hanks, R. J., and Bowers, S. A., "Numerical Solution of Moisture Flow Equation for Infiltration into Layered Soils," Proceedings, Soil Science Society of America, Vol. 26, 1962, pp. 530–534.

Basically, Eq. 8(a) gives the change in pressure head, h, in increment, i, of thickness Δz, during an increment of time, Δt, which is equal to the average flow into layer i minus the average flow out of i divided by the differential fluid capacity, C. Because this problem pertains only to the drainage cycle, and hysteresis in the P_c - θ relation would not occur, a unique relation was assumed between θ and h to obtain the capacitance C. With estimates of C and K, n equations with n unknowns can be obtained, and, when written in matrix form, result in a tridiagonal matrix. The same programmed method was used to solve these equations as was used by Hanks and Bowers. Values of K and C were held constant over a time increment, but were adjusted for each time increment. The number of increments used in each case was 20.

The time increment, Δt, was variable, and was estimated as

$$(\Delta t)^{j+1/2} = Q/q^{j-1/2} \quad \text{(9)}$$

in which Q = the approximate amount of outflow per time increment, and $q^{j-1/2}$ = the drainage rate for the previous time increment. The value of Q was taken as 0.05 Δz. The hydraulic conductivity, K, at a given θ was estimated using the "forward" Gregory-Newton curvilinear interpolation formula from θ near the saturation value to within five increments of the lower end of the θ table (Wylie[39]). The "backward" Gregory-Newton formula was used for small values of θ. The interpolation equation was a third-order polynomial. The capitance, $C = \Delta\theta/\Delta h$, was calculated from a table of θ versus h, where $\Delta\theta$ was the increment in which the estimated fluid content of each layer fell. The estimated value of C for a time interval was based on an estimate of θ near the end of the time increment using

$$\theta_i^{j+1} = \left[\theta_i^j - \theta_i^{j-1}\right] B + \theta_i^j \quad \text{(10)}$$

in which B = 0.7 or $t/(t + 3.33)$, whichever was greater. The estimate of K_i for a time interval was based on the value of θ_i at the beginning of the time interval. The major change in the program from that used by Hanks and Bowers was in the method of estimating K.

A minor adjustment of total time was required to compensate for the initial boundary conditions imposed for solutions N-4 to N-13. This adjustment consisted of extrapolating the total flow beyond t = 0 to obtain a small time increment that would have been required to desaturate the column to its starting values. A second-order interpolation equation was fitted to the initial outflow data points for this purpose. This time increment was added to the cumulative time and the small amount of initial desaturation (0.01 $L/2$) assumed was added to the cumulative drainage, Q. A similar adjustment was needed for solutions N-1 to N-3 to distribute the initial jump from a fully saturated column to one varying from θ_o at the bottom to approximately θ_o - 0.01 near the top.

RESULTS AND EVALUATION

Comparison of Laboratory and Numerical Solutions.—Outflow data from one of the laboratory experiments using fine sand (G.E. -13) and the numerical solution are illustrated in Fig. 4. The results were quite close except for

[39] Wylie, C. R., Jr., Advanced Engineering Mathematics, McGraw-Hill Book Co., Inc., New York, N. Y., 1960.

about a 10% deviation near 0.2 day. The distribution of fluid pressure is presented in Fig. 5. Close agreement occurred at 4.47 hr and 2.53 days. The general pattern was similar at 0.91 hr, although the absolute values differed. Fluid distribution at four times is presented in Fig. 6. The general desaturation pattern was similar from 0.91 to 2.53 days. The laboratory column apparently had several minor nonuniform layers which were probably caused by joining the packed column section. The column was joined at the 44-cm and 86-cm positions.

The comparisons of data from the three laboratory experiments with the three numerical solutions confirmed the general differential equation of flow (Eq. 7), the numerical equation of flow [Eq. 8(*a*)], and the numerical procedure used. They also demonstrate the applicability of fluid content-fluid pressure,

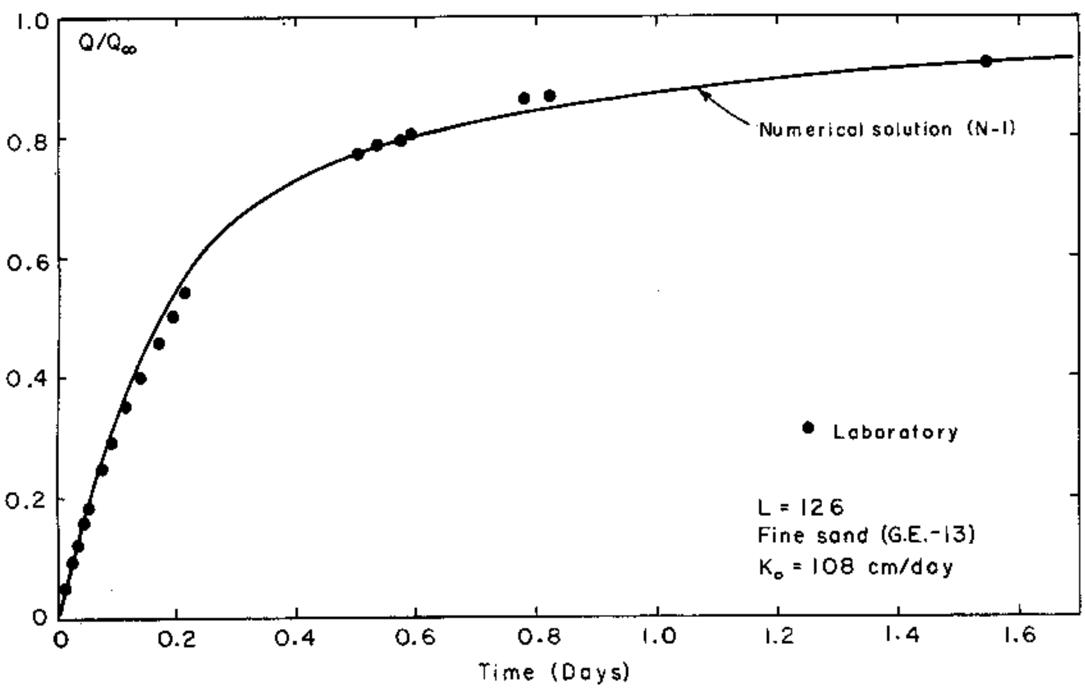


FIG. 4.—COMPARISON OF LABORATORY OUTFLOW DATA WITH THE NUMERICAL SOLUTION

and relative permeability-fluid pressure relationships, independently determined under steady-state conditions on samples of the porous media, for solving nonsteady-state flow problems. The evaluations and interpretations in the remainder of this report are based on the results from the numerical solutions. Actually, the numerical solutions provide more reliable data for interpretation, because irregularities or nonhomogeneities that occur in laboratory columns are not present, and the pressure and fluid content distributions are probably more exact than could have been determined with the laboratory apparatus used.

Initial Drainage Rates.—The assumption of a linear relationship for h versus θ for the increment θ_o - (θ_o - 0.01) leads to unrealistic initial flow conditions. In the numerical solution, the column must desaturate uniformly from $\theta = \theta_o$ at the bottom of the column to $\theta = \theta_o$ - 0.01 at the point that in a real

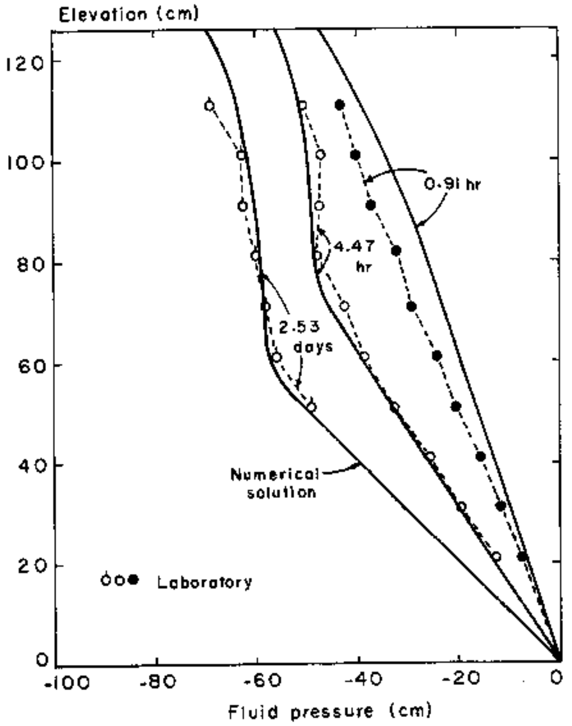


FIG. 5.—COMPARISON OF LABORATORY FLUID PRESSURE DISTRIBUTION WITH THE NUMERICAL SOLUTION

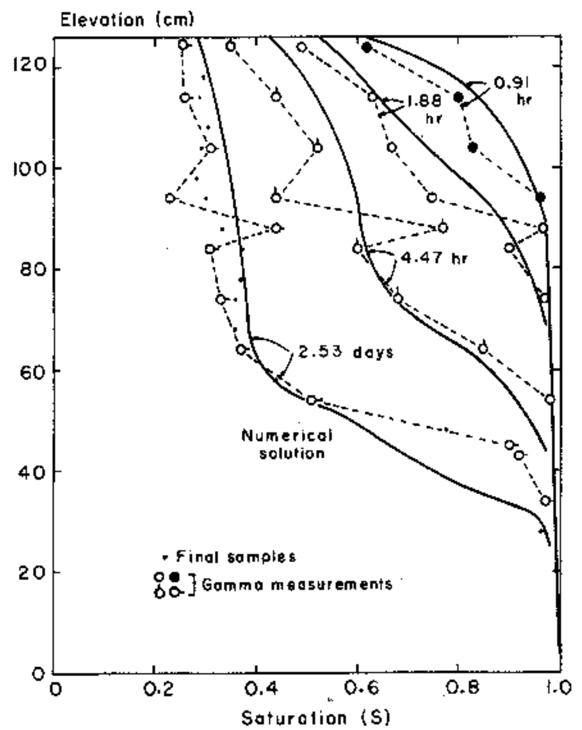


FIG. 6.—COMPARISON OF LABORATORY FLUID DISTRIBUTION WITH THE NUMERICAL SOLUTION

column would represent the saturated front (see Fig. 6). The saturated front in a real column would be about z = 90 cm after 0.9 hr of drainage. The numerical solution indicates a linear desaturation from S = 0.97 at 90 cm to S = 1.0 at 0 cm. Thus, although a second-order polynomial interpolation equation was used to project the outflow rate to t_0, the numerical solution is probably the least accurate for initial flow rates, q_0. The differences were

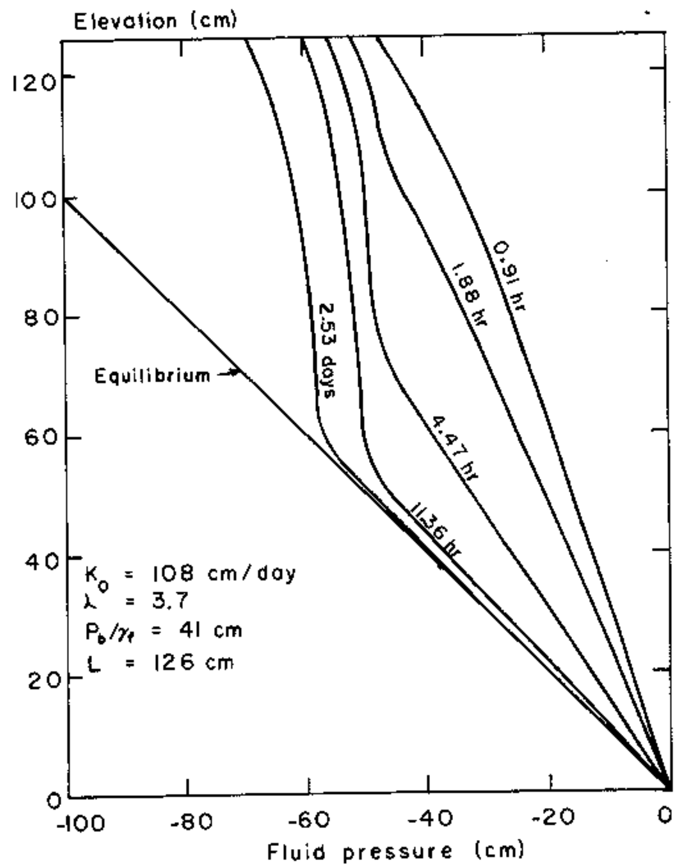


FIG. 7.—DISTRIBUTION OF FLUID PRESSURE DURING DRAINAGE OF A COLUMN OF SILT LOAM

greatest for small column lengths relative to $-h_1$, $(-L/h_1 < 2)$. In general, there was good agreement between the numerical solutions and

$$q_0 = K_0 \frac{dH}{dz} = K_0 \frac{(L + h_1)}{L} \qquad (11)$$

Pressure Distribution With Time.—Fluid pressure at 7 different times in relation to elevation above the outflow position is presented for a column 300 cm in length in Fig. 7. The material had a saturated hydraulic conductivity of 21.7 cm per day and a pore-size distribution index of 1.82.

The fluid pressure gradient, dh/dz, at a given time initially is essentially constant. Likewise, the hydraulic gradient, dH/dz, will be constant. When the fluid pressure near the top of a long column ($L >> P_b/\gamma_f$) approaches a value where $K << K_0$, then the rate of change in the fluid pressure near the surface decreases and the fluid pressure distribution becomes nonlinear except in the saturated zone, Fig. 7. The pressure gradient, dh/dz, with coarse-textured materials that were used, rapidly approached zero ($dH/dz \approx 1$) because the hydraulic conductivity became small in relation to K_0 as the column desaturated, thus greatly impeding further desaturation. The pressure gradient did not exhibit this characteristic change as distinctly for a finer textured material, because the hydraulic conductivity does not decrease as sharply as the column desaturates, Fig. 7. The pressure distribution below the saturated front ($S \approx 0.97$) remained essentially linear, with dh/dz gradually approaching - 1.0 with time, as would be expected. The pressure distribution above the critical elevation, z_s, remained essentially linear. The value of z_s is 108 cm for Fig. 7.

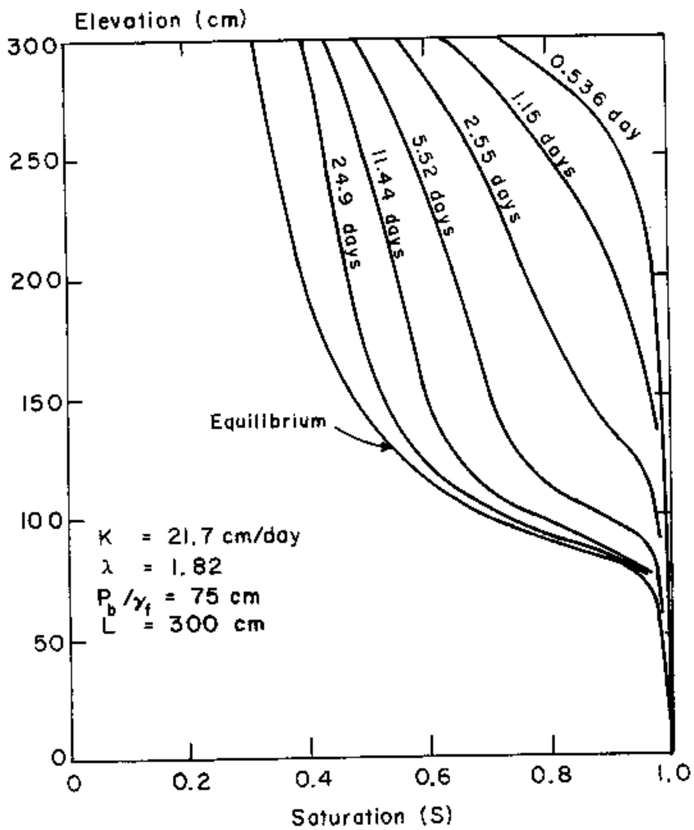


FIG. 8.—DISTRIBUTION OF FLUID CONTENT DURING DRAINAGE OF A COLUMN OF SILT LOAM

Fluid Content Distribution With Time.—Fluid content distribution at 7 different times is presented for the same porous medium and column in Fig. 8.

The desaturation of a long column ($L >> P_b/\gamma_f$) tends to display three general desaturation patterns: (1) An initial, short term, desaturation pattern that is triangular, with the degree of desaturation corresponding to the equilibrium fluid pressure-fluid content (retentivity) curve (the fluid pressure distribution is linear during this period); (2) desaturation after the hydraulic conductivity near the surface has begun to impede the rate of desaturation, but the saturated front is still above the equilibrium position (the desaturation pattern just above the saturated front begins to approach the shape of the

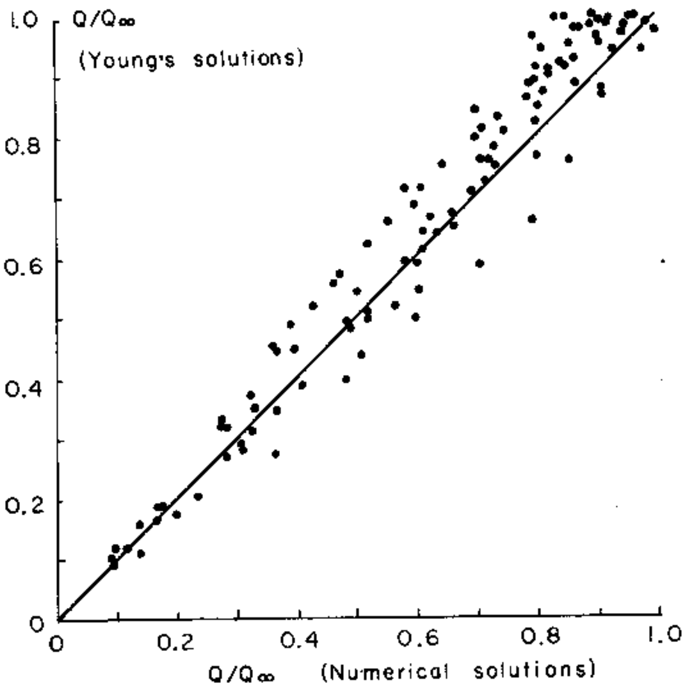


FIG. 9.—COMPARISON OF OUTFLOW PREDICTED USING YOUNG'S EQUATION WITH NUMERICAL SOLUTIONS

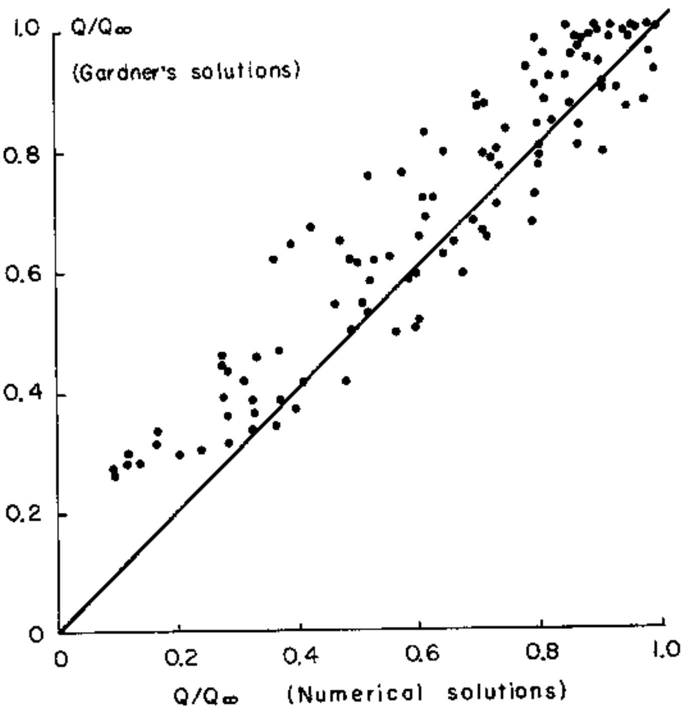


FIG. 10.—COMPARISON OF OUTFLOW PREDICTED USING GARDNER'S EQUATION WITH NUMERICAL SOLUTIONS

TABLE 5.—SUMMARY OF FLOW RATE AND OUTFLOW DATA FROM THE NUMERICAL SOLUTIONS

(a) Fine sand (G.E. - 13) K_0 = 108 centimeter per day P_b/γ_f = 41.0					
No. N-1	L = 126 centimeters		No. N-2	L = 80 centimeters	
Time, in days	q, in centimeters per day	Q/Q_∞	Time, in days	q, in centimeters per day	Q/Q_∞
0.026	81.2	0.096	0.020	—	0.096
0.094	61.0	0.310	0.050	46.7	0.282
0.184	39.2	0.523	0.113	28.6	0.521
0.240	28.2	0.604	0.162	19.7	0.637
0.385	12.2	0.723	0.195	15.3	0.694
0.600	6.08	0.800	0.299	7.7	0.803
1.017	2.99	0.877	0.390	4.8	0.855
1.617	1.99	0.926	0.543	2.9	0.903
2.517	0.91	0.958	1.095	1.1	0.967
3.417	0.91	0.975	1.995	1.1	0.997
(b) Touchet silt loam K_0 = 21.7 centimeters per day P_b/γ_f = 75.0 centimeters					
No. N-4	L = 300 centimeters		No. N-5	L = 140 centimeters	
0.366	13.91	0.091	0.173	10.6	0.137
1.390	8.94	0.278	0.404	5.94	0.276
2.909	5.91	0.473	0.591	5.03	0.369
4.255	4.04	0.588	0.812	4.23	0.464
6.335	2.44	0.701	1.076	3.52	0.555
9.953	1.34	0.812	1.395	2.86	0.647
12.94	0.86	0.867	1.795	2.22	0.737
15.94	0.62	0.905	2.318	1.64	0.825
21.94	0.36	0.953	3.048	1.10	0.910
24.94	0.30	0.970	4.186	0.65	0.988
(c) Volcanic sand K_0 = 700 centimeters per day P_b/γ_f = 16.0 centimeters					
No. N-7	L = 136 centimeters		No. N-8	L = 60 centimeters	
0.0066	485.0	0.117	0.0039	400.0	0.168
0.0176	443.0	0.273	0.0072	336.0	0.283
0.0353	349.0	0.489	0.0149	248.0	0.500
0.0494	219.0	0.610	0.0212	171.0	0.627
0.0869	46.2	0.714	0.0274	115.0	0.709
0.171	21.6	0.799	0.0317	86.0	0.748
0.306	12.9	0.850	0.0458	41.6	0.824
0.524	6.1	0.895	0.0577	29.1	0.859
1.673	3.3	0.955	0.0760	17.7	0.891
7.499	3.3	0.984	0.1051	14.5	0.922
(d) River sand (screened) K_0 = 400 centimeters per day P_b/γ_f = 24.0 centimeters					
No. N-10	L = 126 centimeters		No. N-11	L = 30 centimeters	
0.0116	323.0	0.114	0.0061	80.3	0.328
0.0330	293.0	0.311	0.0070	68.3	0.369
0.0571	252.0	0.509	0.0081	64.0	0.408
0.0689	225.0	0.599	0.0120	47.3	0.566
0.0877	175.0	0.707	0.0172	42.2	0.678
0.1065	119.0	0.795	0.0238	27.7	0.803
0.1407	55.0	0.866	0.0342	17.3	0.910
0.2023	15.0	0.910	0.0472	17.3	0.978
0.3732	9.8	0.953	0.0601	17.3	1.000
0.8782	2.9	0.9999			
(e) Weld silty clay loam K_0 = 49.0 centimeters per day P_b/γ_f = 57.0 centimeters					
No. N-12,	L = 200 centimeters		No. N-13	L = 120 centimeters	
0.174	—	0.168	0.086	—	0.202
0.421	21.53	0.332	0.168	—	0.325
0.791	15.52	0.529	0.294	11.85	0.490
1.004	12.14	0.613	0.402	9.71	0.587
1.473	6.93	0.734	0.501	8.13	0.661
2.015	4.09	0.811	0.620	6.50	0.731
2.430	2.79	0.848	0.771	5.01	0.801
4.003	1.18	0.920	0.972	3.53	0.868
5.422	0.78	0.954	1.266	2.28	0.932
6.422	0.57	0.970	1.746	1.25	0.988

equilibrium retentivity curve during this period, and the fluid pressure distribution is no longer linear); and (3) slow desaturation of the entire column above the saturated and transition zone that nearly parallels the equilibrium retentivity curve—the fluid pressure gradient, dh/dz, below the saturated front is approaching the equilibrium gradient of -1, but may approach and remain near zero ($dH/dz \approx 1.0$) above the critical elevation, z_3, for some time, depending on the hydraulic conductivity-fluid content characteristics.

These characteristic changes in the pressure and fluid distribution with time illustrate the difficulty of representing the drainage from a column with a single algebraic expression. Laikopoulos[22] recently concluded: "The prediction of the one-dimensional transient flow problems through soils cannot be given by a single algebraic equation. The equation that governs these processes is a partial differential equation which furthermore has strong nonlinear terms and cannot by any means be reduced to a simple expression."

Drainage Rates.—A summary of drainage rates, q, and outflow data Q/Q_∞, for 10 of the 13 numerical solutions is presented in Table 5. These data illustrate the magnitude of error that may be encountered when assuming a constant specific yield. A comparison between Youngs' and Gardner's approximate solutions is presented in Figs. 9 and 10. A summary of the exponential coefficients used in Figs. 9 and 10 is presented elsewhere.[30]

Youngs' equation consistently was reasonably accurate for low values of Q/Q_∞, especially for the coarser textured materials. Above 0.6, Youngs' equation generally overestimated Q/Q_∞. This is in agreement with Youngs' data. The estimates generally were less reliable for $L < 2\,P_b/\gamma_f$.

The major problem encountered with Gardner's equations is in obtaining a suitable value for $\bar{D}$. Gardner evaluated Eq. 3(a) by calculating or selecting a value for $\bar{D}$ to fit the data. Estimates using $\bar{D} = K_0 L^2/Q_\infty$ were completely unreasonable. When K near the surface becomes appreciably smaller than K_0 (actually for coarse textured materials and $L > 2\,P_b/\gamma_f$, $K \rightarrow 0$), use of an average conductivity, $\bar{K}$, was proposed. In Fig. 10, an arithmetic average of K_0 and K at the equilibrium fluid pressure at the surface of the column was used for $\bar{K}$. The use of $(\alpha_1)^2$, or the ratio of the impedance in the fringe to the remainder of the column, improved the estimates using Gardner's equation with $\bar{D}$ = a constant; however, the results were generally much too large for small values of Q/Q_∞, especially for the longer columns (see Fig. 10).

One objective of an approximate solution is to predict the total drainage in a given time period with reasonable accuracy. Practical applications of Gardner's equation with $\bar{D}$ = a constant would require using the adjustment for the fringe, z_1, and an improved technique for selecting an average value of K. Improved estimates can be made using a variable value for D in Eq. 3(a) and using numerical solutions derived for a variable D. However, for many practical applications in drainage, the estimates obtained using Youngs' equation are sufficiently accurate and easier to obtain. In addition, only the parameters K_0, P_b/γ_f or h_1, and Q_∞ are required. An estimate of Q_∞ can be made if, in addition, the parameters λ, S_r, and ϕ are known for the porous materials involved. The rate of change in fluid pressure with desaturation, $dh/d\theta$, near the critical elevation, z_3, or in the zone above z_3 appears to influence the rate of change of τ with time. A modification of Gardner's equation may improve estimates of outflow during the period of secondary drainage.

The numerical solution, N-5, displayed pressure distribution and fluid distribution patterns similar to the numerical solution presented by Liakopoulos,[22] except during the first hour. The drainage rate in his solution decreased in a similar manner, approaching a constant after about 5 hr (K_0 = 37.8 cm per day, h_1 = -80 cm), whereas the drainage rate in N-5 began to level off after 3 days (K_0 = 21.7 cm per day, h_1 = - 74 cm). A major difference between the two numerical solutions appears to be due to hydraulic gradients during the first 1 hr to 2 hr. In Liakopoulos' solution, the fluid pressure at the surface did not reach -80 cm until about 30 min after drainage began (highly unlikely in a real column). Also, the hydraulic gradient remained near 1.0 in the lower portion of the column nearly 1 hr, resulting in more rapid initial drainage. The initial drainage rate was projected to 43.2 cm per day at t = 0, which would be unrealistic considering that K_0 = 37.8 Luthin and Miller[29] presented experimental data that showed a sudden decrease in fluid pressure throughout the column as soon as drainage began. They also stated that the fluid pressure must exceed the air-entry value at the surface before drainage can begin in field soils where soil subsidence is negligible. In laboratory experiment E-3, a change in fluid pressure head near the top of the column of 18 cm was recorded within 1 min after free fluid disappeared from the surface. These data indicate that the numerical procedure used by Liakopoulos did not adequately represent the rapid initial changes that occur as soon as drainage begins. In a numerical solution of a drainage problem presented by Remson et al.,[40] the water table was lowered in small steps. Also, initial flow rates and pressure changes were not presented which would have permitted a direct comparison. Remson et al. presented a numerical solution for drainage of a sandy loam soil as the water table was lowered from the surface to a depth of 415 cm. They also included evaporation from the surface. The cumulative drainage, Q/Q_∞, was similar to the solutions presented in this study.

The numerical solutions obtained also were similar to experimental data for five columns presented by Prill, et al.[41] The moisture contents in these columns were calculated using a moisture-tension curve, and no moisture distribution data were presented for the first hour of drainage. A similar comparison between experimental data and a numerical solution was being made about the same time as this study by Watson in Australia.[42] Watson used a fine sand in a 10 cm by 15 cm rectangular column 57 cm in height. The major difference between the studies was in the determination of the hydraulic characteristics for the material and in the computing process. The saturated hydraulic conductivity was determined in the column as in this study. However, the hydraulic conductivity for other water contents were obtained from the drainage column instead of using the hydraulic characteristics of the ma-

[40] Remson, I., Drake, R. L., McNeary, S. S., and Wallo, E. M., "Vertical Drainage of an Unsaturated Soil," Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY1, Proc. Paper 4196, Jan., 1965, pp. 55-74.

[41] Prill, R. C., Johnson, A. G., and Morris, D. A., "Specific Yield—Laboratory Experiments Showing the Effect of Time on Column Drainage," U. S. Geological Survey Paper 1662-B, 1965.

[42] Watson, Keith K., "Experimental and Numerical Study of Column Drainage," Journal of the Hydraulics Division, ASCE, Vol. 93, No. HY2, Proc. Paper 5120, Mar., 1967, pp. 1-15.

terial determined on independent samples. An iteration process was used in obtaining the numerical solutions.

SUMMARY AND CONCLUSIONS

Thirteen numerical solutions of one-dimensional non-steady-state drainage problems involving five different porous media and two or three different boundary conditions for each were obtained using a high speed digital computer. Experimental data were obtained in the laboratory under conditions comparable to three of the numerical solutions to confirm the numerical procedure used. A pressure transducer and valve manifold were used to measure fluid pressure, and the gamma radiation technique was used to measure fluid distribution in the columns during drainage. The laboratory data verified the numerical procedure being used, even during the more rapid changes that occur when drainage first begins.

The linear fluid pressure distribution in a column just after drainage begins varied from $h = 0$ at the base of the column to $h = h_1$, or the pressure at which rapid desaturation begins at the surface. The fluid pressure at the surface changes initially at a rate related to the minimum value of $dh/d\theta$, or the reciprocal of the maximum volumetric differential fluid capacity, $(d\theta/dh)_{max}$.

The pressure distribution becomes nonlinear after a small time except below the descending saturated front. When t becomes large, the pressure gradient, dh/dz approaches -1.0, or the equilibrium value, below the saturated front. Above the saturated front the pressure gradient also approaches -1.0, but after a considerably longer period of time, depending on the hydraulic properties of the porous media and the boundary conditions.

The rate of drainage from columns can be predicted with reasonable accuracy using Youngs'[7] equation up to $Q/Q_\infty \approx 0.6$. An analysis of the draining column indicated that as the saturated front approached the equilibrium position the flow regime may change. Distinct changes in the flow regime occurred in several of the numerical solutions. The change in flow regime coincides approximately with the point at which Youngs' equation begins to overestimate the outflow volume.

Predicted outflow values using Gardner's[26] equation with $\bar{D}$ = a constant were not as accurate as those obtained using Youngs' equation. A change in the proposed procedure for calculating $\bar{D}$ would probably improve the estimates using Gardner's equation. Also, a simplified procedure for using a variable D instead of $\bar{D}$ = a constant could improve the solutions obtained and still retain the relative simplicity of the equation.

The results summarized in this study are presented in more detail elsewhere.[30]

ACKNOWLEDGMENTS

This paper is a contribution from the Northwest and Northern Plains Branches, Soil and Water Conservation Research Division, Agricultural Research Service, U S. Department of Agriculture, with the Colorado and Idaho Agricultural Experiment Stations cooperating.

APPENDIX.—NOTATION

The following symbols are used in this paper:

$C = d\theta/dh$ = volumetric differential fluid capacity;
$\overline{D} = K_0 L^2/Q_\infty$ = mean diffusivity (see Gardner's equation);
$h = P_w/\gamma_f$ = fluid pressure head;
h_c = critical fluid pressure head;
h_1 = fluid pressure head when rapid desaturation begins;
h_3 and h_4 = fluid pressure heads at specific elevations in the columns;
$H = P_w/\gamma_f + z = h + z$ = hydraulic head;
$K = f(\theta)$ = unsaturated hydraulic conductivity;
K_0 = saturated hydraulic conductivity;
$k_r = K/K_0$ = relative hydraulic conductivity;
L = length of soil column;
P_b = bubbling pressure, approximately the minimum value for P_c on the drainage cycle at which a continuous nonwetting phase exists in the porous medium;
P_c = capillary pressure (the pressure difference $P_{nw} - P_w$);
P_{nw} = pressure of the nonwetting fluid;
P_w = pressure of the wetting fluid;
Q = total outflow volume in time, t, used as volume per unit area;
Q_∞ = total outflow volume as $t \to \infty$, used as volume per unit area;
Q_1 = total primary drainage per unit area;
Q_2 = total secondary drainage per unit area;
q = flow rate across a unit area of a porous medium;
q_0 = initial flow rate;
$S = \theta/\phi$ = saturation, ratio of the volume of wetting fluid to the volume of interconnected pore-space in the porous medium;
$S_e = S - S_r$ = effective saturation;
S_r = residual saturation, saturation at which K is assumed to be zero for calculation purposes;
t = time;
t_1 = transition time between primary and secondary drainage;

z = vertical coordinate, reference datum = 0 at the base of the column where P_c/γ_f = 0 during drainage;

Δ = denotes a difference or increment;

$(\alpha_1)^2$ = ratio of impedance of the capillary fringe to the remainder of the column;

γ_f = weight of fluid per unit volume;

η = exponent in the equation $k_r = (P_b/P_c)^\eta$;

θ = volumetric fluid content;

θ_0 = value of θ when the porous medium is saturated;

λ = the exponent in the equation $S_e = (P_b/P_c)^\lambda$;

τ = exponent in the equation $Q/Q_\infty = 1 - e^{-\tau}$; and

ϕ = porosity, ratio of the volume of pore space to total volume of the porous medium.

ERRATA for "Nonsteady-State Drainage from Porous Media"
by Marvin E. Jensen and R. John Hanks

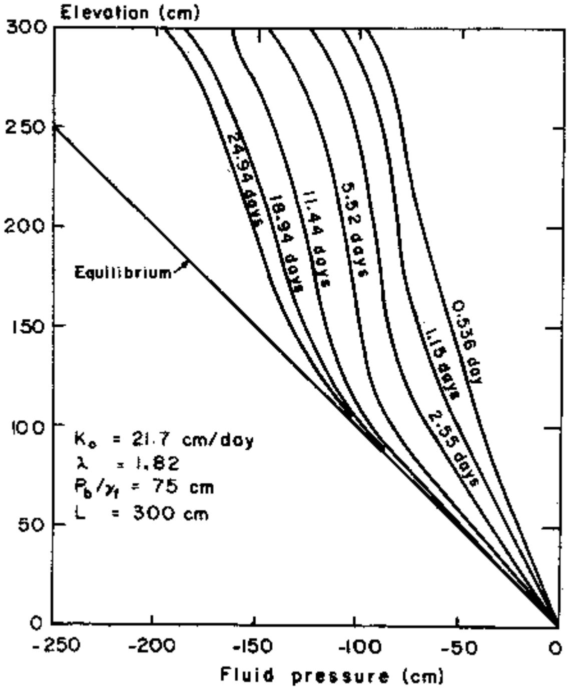
Proc. ASCE, J. Irrig. & Drain. Div.,
IR 3, Paper 5444, September 1967

Page 217: γ_t should be γ_f

Page 219: Negative sign was omitted from equation 8a (between the major terms within brackets).

Page 223: The wrong drawing was used in FIG. 7. The correct drawing is presented below.

Page 227: Line 10 "Laikopoulos" should read "Liakopoulos."



5444 NONSTEADY-STATE DRAINAGE FROM POROUS MEDIA

KEY WORDS: drainage; fluid flow; nonsteady-state; numerical solutions; porous media; unsaturated flow

ABSTRACT: The validity of numerical solutions of nonsteady-state, one-dimensional, vertical drainage problems was evaluated by comparison with experimental data. Experimental data included simultaneous measurements of fluid pressure using a pressure transducer, and fluid content using gamma radiation. The results of 13 numerical solutions involving five porous media and two or three boundary conditions for each are summarized. Several approximate solutions to the vertical drainage problem were evaluated and the limitations of each discussed. The flow rate was related to the position of the saturated front during primary drainage. During secondary drainage, the change in flow rate was not dependent on the recession of the saturated zone, but was related to the unsaturated hydraulic conductivity and the differential fluid capacity above the saturated zone for the porous material and fluid involved. Characteristic changes in fluid pressure distribution with time are summarized.

REFERENCE: Jensen, Marvin E., and Hanks, R. John, "Nonsteady-State Drainage from Porous Media," Journal of the Irrigation and Drainage Division, ASCE, Vol. 93, No. IR3, Proc. Paper 5444, September, 1967, pp. 209-231.